This is a study guide for Exam 1. You are expected to understand and be able to answer mathematical questions on the following topics.

Chapter 26

A capacitor has charge +Q and –Q stored on two charge conductors, and a potential difference of ΔV between the conductors. The **capacitance** is found by

$$C = \frac{Q}{\Delta V}$$

where C is always constant (thus the magnitudes of ΔV and Q are used). Capacitance is a physical quantity for an object that does not change, regardless of the stored charge or potential applied. Units of capacitance:

$$1F = \frac{1C}{1V}$$

For a parallel plate conductor,

$$C = \frac{\epsilon_o A}{d} \qquad E = \frac{\sigma}{\epsilon_o} \qquad \Delta V = Ed$$

The energy stored on a capacitor is found by

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

This energy could be considered as being stored in the electric field. Since $\Delta V = Ed$ and $C = \epsilon_o A/d$, we can say

$$U = \frac{1}{2}\frac{\epsilon_o A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_o A d)E^2$$

Since this electric energy is within the volume *Ad* of the capacitor, the energy per unit volume is found by

$$u_E = \frac{U}{Ad} = \frac{1}{2}\epsilon_o E^2$$

This expression is valid for any electric field, not just for capacitors.

Parallel Combination

Two capacitors are in **parallel** when they are both attached to the same points in the circuit, and they both have the same electric potential:

$$\Delta V_1 = \Delta V_2 = \Delta V$$

The two capacitors will accumulate a total charge equal to

$$Q_{total} = Q_1 + Q_2$$

This can be used to create an *equivalent capacitor* with capacitance C_{eq}. The equivalent capacitor would act as though the two capacitors were really one, storing a charge Q_{total} across a potential ΔV.

$$C_{eq}\Delta V = Q_1 + Q_2 = C_1\Delta V_1 + C_2\Delta V_2 = (C_1 + C_2)\Delta V$$

Thus, the **equivalent capacitance** can be found by summing the capacitance of capacitors in parallel.

$$C_{eq} = C_1 + C_2 + \cdots$$

Series Combination

When capacitors are in series, the charge removed from one capacitor is sent to the next. Thus, the change in charge for each capacitor is the same:

$$Q_1 = Q_2 = Q$$

An equivalent capacitor would then have $+Q$ on one plate, and $-Q$ on the opposite. The total voltage across the capacitors in series is shared, so

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

To find the equivalent capacitor, we can say

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{Q}{C_{eq}}$$

Since $Q_1 = Q_2 = Q$, the equivalent capacitance would be

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Dielectrics

A **dielectric** is a nonconducting material such as rubber of glass. Each material has a **dielectric constant**, a dimensionless factor $\kappa > 1$ which is used to find how much a material affects the voltage across a capacitor if it is placed between the plates. If a dielectric is placed between the plates of a capacitor, the voltage across the capacitor will change. The new voltage is found by

$$\Delta V = \frac{\Delta V_o}{\kappa}$$

However, the stored charge on the capacitor does not change. Thus, the capacitance must.

$$C = \frac{Q_o}{\Delta V} = \frac{Q_o}{\Delta V/\kappa} = \kappa\frac{Q_o}{\Delta V_o} = \kappa C_o$$

For a parallel plate capacitor, the area and distance between the plates doesn't change. Thus,

$$C = \kappa\frac{\epsilon_o A}{d}$$

Chapter 27

The **current**, or rate of charge flow through an area, is found the amount of charge passing through an area in time dt:

$$I = \frac{dQ}{dt}$$

The unit of current is an **ampere** where $1A = 1\ C/s$. While current can be defined as the movement of positive or negative charges (**charge carriers**) in any direction, the convention is to say current is in the direction of the flow of positive charges. This means that in circuits, current is the opposite direction of the motion of electrons.

In order for the current to exist, an electric potential is applied through a material, which creates an electric field inside the conducting material, causing the charge carriers to move.

Physical Model

If a conducting wire with cross-sectional area A has n charges per volume, each with charge q, the amount of charge within a segment Δx would be

$$\Delta Q = (nA\Delta x)q$$

If the charge carriers move with a velocity of v_d (d is for drift speed), and it takes the charges a time Δt to move the distance Δx, then

$$\Delta Q = (nAv_d\Delta t)q$$

The current can thus be found as a function of the charge carriers by

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nAv_d q$$

Resistance

The **current density** is the current per unit area, or

$$J = \frac{I}{A} = nqv_d$$

and has units of A/m^2. In many materials, the current density is proportional to the electric field by

$$J = \sigma E$$

This is **Ohm's Law**, where σ is a constant (*not the surface charge density*) called the **conductivity** of the material. The electric potential that is applied to create the current along a length l of the wire is

$$\Delta V = El$$

So

$$J = \sigma\frac{\Delta V}{l}$$

Rearranging:

$$\Delta V = \frac{l}{\sigma}J = \left(\frac{l}{\sigma A}\right)I = RI$$

Where R is a constant for the material, called its **resistance**.

$$R = \frac{\Delta V}{I}$$

and has the unit of an **ohm** (Ω), where $1\Omega = 1\text{V/A}$. The inverse of conductivity is **resistivity** ρ:

$$\rho = \frac{1}{\sigma}$$

and has units of ohm·meters ($\Omega \cdot m$). The resistance can then be written as

$$R = \rho\frac{l}{A}$$

The resistivity of a conductor varies with temperature according to

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

where ρ_o is the resistivity of the material at temperature T_o, and α is the **temperature coefficient of resistivity.** Because the resistance is proportional to the resistivity,

$$R = R_o[1 + \alpha(T - T_o)]$$

Electrical Power

When the electrons move through a resistor, energy is transferred from the motion of the electrons to the atoms in the resistor, which heats the resistor up. Power (P) is the rate at which energy is delivered to the resistor:

$$P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Chapter 28

Resistors: Series Combination

If two resistors are in series, they are in line on the same wire. If charge passes through one, it has to pass through the other, so the current is the same in both resistors.

$$I = I_1 = I_2$$

Because they are in line, the total drop in voltage across the resistors is shared between them as

$$\Delta V = \Delta V_1 + \Delta V_2 = I_1 R_1 + I_2 R_2$$

An equivalent resistor would have the total voltage across both resistors, follow the relation

$$\Delta V = I R_{eq} = I_1 R_1 + I_2 R_2$$

So we can say the equivalent resistance will be the sum of the other resistors in series.

$$R_{eq} = R_1 + R_2 + \cdots$$

Parallel Combination

If the resistors are set up in parallel, they each have the same potential difference across them.

$$\Delta V = \Delta V_1 = \Delta V_2$$

The charges flowing through the system may flow through only one of the resistors in parallel. The total current will then be the sum of the currents flowing through all of the resistors.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

The current flowing through an equivalent resistor would then be

$$I = \frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

Therefore, for series in parallel, the equivalent resistance would then be

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

This means the equivalent resistance is always less than the smallest resistor.

Kirchhoff's Rules

Two principles in Kirchhoff's Rules:

1. The sum of currents in and out of a junction is zero.

$$\sum_{junction} I = 0$$

This is because more charges can't enter a junction than leaves, or vice versa, because this would require charge to accumulate or deplete. Current entering a junction is written as $+I$ while current leaving a junction is $-I$.

2. The sum of potential differences across all elements around any closed circuit loop is zero.

$$\sum_{closed\ loop} \Delta V = 0$$

This is due to conservation of energy. If a charge moves through the circuit, it must have the same potential energy if it returns to where it started. It increases in potential energy when the charge passes through a battery from the negative to positive terminal (the positive terminal of a battery in a circuit diagram is drawn with a longer line).

When following a path around a current loop, follow these rules:

1. If you pass through a resistor in the direction of the current, then $\Delta V = -IR$.
2. If you pass through a resistor opposite the direction of the current, then $\Delta V = IR$.
3. If you pass through a battery from negative to positive terminals, then $\Delta V = \mathcal{E}$.
4. If you pass through a battery from positive to negative terminals, then $\Delta V = -\mathcal{E}$.

Equations can be made following Kirchhoff's Rules to find the characteristics of a circuit (currents, resistances, voltages). The number of equations must be at least as many as the unknowns.

Charging an RC Circuit

A circuit combination of a resistor and a capacitor is called an **RC Circuit**. Assume the capacitor is initially uncharged. When power is applied, the potential across the capacitor increases as the capacitor charges. Once the capacitor is charged to where the potential across it is the same as the power supply, the current in the circuit stops. Using the loop rule,

$$\mathcal{E} - \frac{q}{C} - IR = 0$$

However, while the capacitor is being charged, there is a current and q and I are both functions of time. At time $t = 0$, the charge on the capacitor is zero ($q = 0$), so the *initial current* is also the *maximum current* where

$$I_{max} = I_o = \frac{\mathcal{E}}{R}$$

When the capacitor is charged, the current will be zero. The *maximum charge* Q on the capacitor will then be

$$Q_{max} = C\mathcal{E}$$

The charge on the capacitor as a function of time is found by

$$q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

As for the current in the system, it is found by

$$I(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}} = I_{max}e^{-\frac{t}{RC}}$$

These fit our limits for initial current (when $t = 0$) and maximum charge (when $t = \infty$) on the capacitor. The quantity RC is called the **time constant** τ, so

$$\tau = RC$$

This is the amount of time for the current to decrease to e^{-1} of its initial value (after time τ, $I = 0.368I_i$), and has units of seconds. This means for charging,

$$I(t) = I_{max}e^{-\frac{t}{\tau}}$$

and

$$q(t) = Q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Discharging an RC Circuit

Assume the capacitor is charged to a maximum charge Q, and the battery is removed from the circuit. If we allow the capacitor to discharge, the current will go the other way around the circuit than it did before. Using the loop rule,

$$-\frac{Q}{C} - IR = 0$$

So the initial current is

$$I = -\frac{Q}{RC}$$

The charge on the capacitor is found by

$$q(t) = Q_{max}e^{-\frac{t}{RC}} = Q_{max}e^{-\frac{t}{\tau}}$$

Differentiating with respect to time, we find the current in the circuit as a function of time is:

$$I(t) = \frac{dq}{dt} = -\frac{Q}{RC}e^{-\frac{t}{RC}} = -I_{max}e^{-\frac{t}{\tau}}$$

The negative just shows the current is going the opposite direction as before, and the time constant is again RC.