This is a review of the topics will be covered in Exam 1.

Chapter 16

Electric Force

Electric charges: The **law of conservation of electric charge** states that the net amount of electric charge produced in any process is zero, and no net electric charge can be created or destroyed. Electric charge is due to the subatomic particles **protons** (positively charged, $q_p = 1.602 \times 10^{-19} C$) and **electrons** (negatively charged, $q_e = -1.602 \times 10^{-19} C$). Opposite charges attract, like charges repel. **Conductors** allow electrons to move through them, **insulators** do not.

Charging objects: You can *charge by conduction* by touching two objects together. This will charge the objects by either one of them removing or adding electrons to the other. *Charging by induction* is when a charged object gets near a conductor, and it either attracts or repels the free electrons in the conductor. This will result in a charge separation so the two objects are attracting, but the net charge in the conductor is zero unless charges are removed.

The magnitude of the force two charged objects exert on each other is found by **Coulomb's law**:

$$F_e = k\frac{Q_1 Q_2}{r^2}$$

The constant k_e is Coulomb's constant where $k = 9 \times 10^9 N \cdot m^2/C^2$. Coulomb's law gives the magnitude of the force between two **point charges**, with the direction between along the line between the charges. If the charges are the same sign, the force is repulsive. If they are opposite, it's attractive. The magnitude of the force that Q_1 exerts on Q_2 is the same as the magnitude of the force that Q_2 exerts on Q_1.

The **permittivity of free space** ($\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$) is the measure of resistance that is encountered when forming an electric field in a medium. It relates to how well a material "permits" an electric field. It is relative to k_e by

$$k = \frac{1}{4\pi\epsilon_0}$$

The Coulomb force is for one charge pair. If there are multiple charges, the force needs to be found for each pair independently and then they can be summed together. The **principle of superposition** says "the net force on any one of the charges will be the vector sum of the forces on that charge due to each of the other charges." The net force on a charge is thus found by:

$$\vec{\boldsymbol{F}}_{\boldsymbol{net}} = \vec{\boldsymbol{F}}_{\boldsymbol{12}} + \vec{\boldsymbol{F}}_{\boldsymbol{13}} + \cdots$$

In order to sum the vectors, follow these steps:

1. Find the *magnitude* of the forces on a particle by each of the other charges.
2. Break the magnitudes down in to $\hat{x}$ and $\hat{y}$ components.
3. Sum the $\hat{x}$ components together and $\hat{y}$ components together to get the net force in those directions.
4. You can find the magnitude and direction of the net force if requested. The magnitude is found by $F = \sqrt{F_x^2 + F_y^2}$ and the direction can be found by $tan\,\theta = F_y/F_x$.

Note: It is also important to draw diagrams to keep track of whether the forces are in the positive or negative $\hat{x}$ and $\hat{y}$ directions.

Electric Field

A charge creates an **electric field**, $\vec{\boldsymbol{E}}$. The electric field will exert a force on a second charge if it is present, but the field exists whether a second charge is present or not. The magnitude of the electric field created by the **source charge** (Q) can be probed by using a **test charge** (q). The electric field is then found by

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{q}$$

where q is the magnitude of the test charge. By multiplying the electric field at a point by the charge of a test particle put at that point, we can find the force the electric field exerts on that particle.

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$$

If the test charge is positive, the force exerted on it will be the same direction as the electric field. A negatively charged particle would feel a force in the opposite direction. For a single point charge:

$$E = k\frac{Q}{r^2}$$

We use **electric field lines** to visualize the electric fields. It helps see what the electric field is for an entire area. Some properties:

1. The electric field is tangent to the lines at any point.
2. The number of lines per area is proportional to the magnitude of the electric field. The denser the lines, the stronger the field.
3. Lines begin at positive charges or infinity, and end at negative charges or infinity.
4. Field lines do not cross.
5. The number of field lines coming from or going to the particles is proportional to the charge on the particle (i.e. a particle with charge $2q$ has twice as many lines as $1q$).

The electric field inside a conductor in electrostatic equilibrium is zero. If there is a net charge on a conductor, it distributes itself on the outer surface. If (and only if) there is a net charge Q inside a cavity inside the conductor, an equal amount of charge but with opposite sign –Q will distribute itself on the inner surface of that cavity. A charge Q will then distribute itself on the outer surface of the conductor.

Chapter 17

Potential Energy

We can define a potential energy for the electric field. The change in potential energy between two points is opposite the work done by the electric force when moving a charged particle between those points.

$$\Delta PE = -W$$

The electric field is doing work on the particle by accelerating it and moving it a distance d. The work done can be found by

$$W = Fd = qEd$$

so

$$\Delta PE = -qEd$$

The change in potential energy due to the electric field moving a particle is negative because the electric force will move the particle from high to low potential energy. The energy is converted into kinetic energy.

Electric Potential

The change in potential energy depends on the sign of the charge. It is useful to define an electric potential V (or just "potential") that will not depend on the test charge. The electric potential at a point a is found by

$$V_a = \frac{PE_a}{q}$$

The units for electric potential is a **volt**, which is defined as $1V = 1J/C$. However, when discussing potential energy, it is only meaningful if we are talking about a *difference* between two points. So, we define

$$V_{ba} = V_b - V_a = \frac{PE_a - PE_b}{q} = -\frac{W_{ba}}{q}$$

This electric potential only depends on the source charges creating the electric field. A positive charge moves from high to low potential, while a negative charge moves from low to high potential. The change in potential energy is found by

$$\Delta PE = q\Delta V$$

Chapter 17.2: Electric Potential and Electric Fields

While an electric field is a vector, electric potential is a scalar. However, the two concepts are connected. The work done by a field to move from point a to point b is

$$W = -qV_{ba} = Fd = qEd$$

where d is the distance between points a and b. We can thus say

$$V_{ba} = -Ed$$

or

$$E = -\frac{V_{ba}}{d}$$

The units for an electric field can also be written as V/m. The minus sign indicates that the electric field is pointed in the direction of decreasing electric potential. **Equipotential lines** are a set of points where the electric potential is the same. The potential difference between any two points on an equipotential line is zero. The electric field is always perpendicular to the equipotential surface. Because the electric field inside a conductor is zero everywhere, the electric potential inside a conductor has to be constant.

For the electric potential around a point source, we define zero potential at infinity. This results in the potential at a distance r from a point charge being

$$V = k\frac{Q}{r}$$

Here, V is the *absolute potential* at a distance r from a charge Q, where $V = 0$ at $r = \infty$. The potential near a positive charge is large and positive, and decreases to zero at infinity. The potential near a negative charge is negative and increases to zero at infinity. If there are multiple point charges, the potential at a point is the sum of the potentials due to each of the point charges.

Capacitance

A **capacitor** can store electric charge. A simple capacitor consists of a pair of parallel plates with area A separated by a distance d. If a voltage is applied across the capacitor, the plates become charged until the plates have a voltage across it equal to the voltage applied. The **capacitance** is how much charge will be stored for a given voltage:

$$C = \frac{Q}{V}$$

Rearranging, we can say that the charge that will be stored on a capacitor is

$$Q = CV$$

The units of capacitance is a **farad** (F). The capacitance is a constant for an object, not depending on Q or V. For a capacitor, the capacitance is found by

$$C = \epsilon_0 \frac{A}{d}$$

where $\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$ is the permittivity of free space.

A capacitor stores electric potential energy equal to

$$PE = \frac{1}{2} QV$$

Because $Q = CV$, we can also say

$$PE = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

Chapter 18

Electric Current

When a battery is connected to a circuit, charge is able to flow as an **electric current**. Current is the amount of charge passing through the wire per unit time, or

$$I = \frac{\Delta Q}{\Delta t}$$

Current is measured in coulombs per second, with the unit of **amperes** $(1A = 1C/s)$.

There can only be current if there is a **complete circuit**. The current is the same everywhere along a single circuit. If the circuit is broken, it is an **open circuit**, and there will be no current.

Resistance

Current is directly proportional to the electric potential. If you double the voltage, you double the current. The constant **resistance** for an object is found by

$$R = \frac{V}{I}$$

If a higher voltage is required to achieve a certain current, the resistance is higher. If a higher current is created by a certain voltage, the resistance is lower. **Ohm's law** is more commonly written as

$$V = IR$$

Resistance has a unit of **ohm** where $1\Omega = 1V/A$. The resistance of an object such as a wire goes as

$$R = \rho \frac{L}{A}$$

where ρ is the resistivity with units of $\Omega \cdot m$, L is its length, and A is the cross-sectional area.

Electric Power

Electric energy from current is transformed into useable energy such as mechanical, thermal, or light. The power going into the circuit is found by

$$P = IV$$

Power has units of a **watt** (1W=1J/s). We can substitute V=IR and use any of

$$P = IV = I^2R = \frac{V^2}{R}$$

Alternating Current

The current created by a battery is **direct current** (DC) while the current in a wall is an **alternating current** (AC). The current reverses direction continuously in a sinusoidal manner. The voltage produced can be represented by the function

$$V = V_0 \sin(2\pi f t) = V_0 \sin(\omega t)$$

where V_0 is the **maximum (peak) voltage**, f is the frequency in Hz, and ω is the angular frequency where $\omega = 2\pi f$. In the US, $f = 60Hz$. If the voltage is applied across a resistor, the current is found by

$$I = \frac{V}{R} = \frac{V_0}{R}\sin(\omega t) = I_0 \sin(\omega t)$$

where I_0 is the **maximum (peak) current**.

The average of $\sin^2(\omega t) = 1/2$, so the **average power** going to a resistor is

$$P = \frac{1}{2} I_0^2 R = \frac{1}{2}\frac{V_0^2}{R}$$

We can also use the **root-mean-square** (rms) value of the current or voltage

$$I_{rms} = \frac{1}{\sqrt{2}} I_0$$

$$V_{rms} = \frac{1}{\sqrt{2}} V_0$$

We can thus say the average power is

$$\bar{P} = I_{rms}V_{rms} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

Chapter 19

Resistors in Series

When resistors are connected end to end, they are in **series**. The same current passes through all resistors in series. Their voltages sum up to be the voltage across the entire circuit:

$$V = V_1 + V_2 + V_3 \ldots = IR_1 + IR_2 + IR_3 \ldots$$

We can find an *equivalent resistor*, R_{eq}, that would draw the same current from the same voltage:

$$V = IR_{eq}$$

in which case

$$R_{eq} = R_1 + R_2 + R_3$$

Their resistances sum to find the equivalent resistor.

So, for resistors in series:

1. the voltages across each resistor adds to be the total voltage across all of the resistors.
2. the current is the same for all resistors.
3. the equivalent resistance is the sum of the resistors in series.

Resistors in Parallel

If resistors are all attached across the same terminal, they are in **parallel**. In this case, the voltage across each of the resistors is the same, and the total current is the sum of the currents through the resistors.

$$I = I_1 + I_2 + I_3$$

We can find an equivalent resistor by using

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

This means the equivalent resistance is found by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

So, for resistors in parallel:

1. the voltages across all of the resistors is the same.
2. the total current is the sum of the currents through each of the resistors.
3. the equivalent resistance is found by the inverse of the sum of the inverses of the resistances.

Kirchhoff's Rules

Analyzing circuits in parallel and series is not good enough if the circuit is too complicated. Kirchhoff's rules are used to analyze more complicated circuits:

1. **Junction rule** – the sum of the currents entering a junction equals the sum of the currents leaving that junction.
2. **Loop rule** – the sum of the changes in potential around any closed path of a circuit must be zero.
   a. If you pass through a resistor in the direction of the current, then $\Delta V = -IR$.
   b. If you pass through a resistor opposite the direction of the current, then $\Delta V = IR$.
   c. If you pass through a battery from negative to positive terminals, then $\Delta V = \mathcal{E}$.
   d. If you pass through a battery from positive to negative terminals, then $\Delta V = -\mathcal{E}$.

Capacitors in Series

For capacitors in series, each capacitor will have the same charge stored. This is because the charge removed from one capacitor will be moved to the next. An equivalent capacitor would replace the entire series, so $V_{eq} = V$.
An equivalent capacitor, C_{eq}, would thus store

$$Q = C_{eq}V$$

The total voltage would also be the sum of the voltages across each capacitor, which means

$$V = V_1 + V_2 + V_3$$

which leads to

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

This means

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

So, for capacitors in series,

1. the voltages sum to the total voltage across all of the capacitors.
2. each capacitor will store the same amount of charge.
3. the equivalent capacitance would be the inverse of the sum of inverses of the capacitances.

Capacitors in Parallel

If capacitors are in parallel, the voltage across them is the same. If we want to create an equivalent capacitor, C_{eq}, the charge stored on it would be the sum of the charge stored on the capacitors in parallel.

$$Q = Q_1 + Q_2 + Q_3$$

We can thus say

$$C_{eq}V = C_1V + C_2V + C_3V$$

This means

$$C_{eq} = C_1 + C_2 + C_3$$

The equivalent capacitance would be the sum of the capacitances of the capacitors in parallel. So, for capacitors in parallel:

1. the voltages are all the same.
2. the equivalent capacitor would store the sum of the charges on the capacitors.
3. the equivalent capacitance would be the sum of the capacitances.

RC Circuits

1. Charging

When an RC circuit is initially closed, the current does not instantly go to its final steady state. The capacitor has to charge, which takes time because as it charges, the voltage across it increases. This acts to slow down the current, which slows how fast the capacitor is charging. The voltage across the capacitor goes as

$$V_C = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

The charge on the capacitor goes as

$$Q = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

We can designate the time constant $\tau = RC$, which is a measure of how quickly the capacitor charges.

2. Discharging

If the power supply is taken out of the circuit, leaving only the resistor and capacitor, the capacitor will discharge. As it discharges, the voltage across the resistor decreases, which decreases the current. Thus it takes time for the capacitor to discharge, with the voltage going as

$$V_C = V_0e^{-\frac{t}{RC}}$$

It has the same time constant $\tau = RC$ as for charging. The charge stored is also decreasing as

$$Q = Q_0e^{-\frac{t}{RC}}$$