This is a review of the topics will be covered in Exam 4.

Chapter 28

The Wave Function

The **wave function** Ψ of a particle represents the information on the position of a particle, such as an electron. The position of the particle depends on a probability distribution, found by Ψ^2. This is due to the wave-particle duality nature of the particle. The electron will show interference when passing through a double slit the same as a photon. It will even interfere with itself if a single electron is sent through the double slit. The probability of where the single electron can be found follows Ψ^2, so if many electrons are sent through the double slit, they will show the double slit pattern.

The Heisenberg Uncertainty Principle

If you want to get information about a particle, such as its momentum or position, you have to interact with it. Information about the position can be found by using light. The accuracy of the position depends on the wavelength of the light used, so the wavelength gives an uncertainty in position:

$$\Delta x \approx \lambda$$

However, as the wavelength of light is decreased to get better position information, the light will have higher energy. When it interacts with the particle, it will transfer the energy, and change its momentum. The error in momentum can be found by using de Broglie's formula

$$\Delta p_x \approx \frac{h}{\lambda}$$

This means that as you decrease your uncertainty in position, you increase your uncertainty in momentum, and vice versa. This results in the **Heisenberg uncertainty principle**, which shows

$$(\Delta x)(\Delta p_x) \geq \frac{h}{2\pi}$$

or, using $\hbar = h/2\pi = 1.055\times10^{-34} J\cdot s$,

$$(\Delta x)(\Delta p_x) \geq \hbar$$

Further, the uncertainty of the energy of a state and the uncertainty of the time in which a particle is in that state are related by

$$(\Delta E)(\Delta t) \geq \hbar$$

Quantum Mechanics

Electrons in an atom are in an electron cloud around the nucleus. The cloud is the probability distribution of the electron. Quantum mechanics provides the information about the state of the electron.

1. *Principal Quantum Number,* n

As found by the Bohr theory, the energy of an electron in a hydrogen atom is found by

$$E_n = -\frac{13.6eV}{n^2}$$

where $n = 1,2,3$ .... This number n is the **principal quantum number** of the electron.

2. *Orbital Quantum Number,* l

The electron also has an **orbital quantum** l, which gives the angular momentum of the electron. This number can range from 0 to $(n-1)$. For example, if $n=3$, l can be 0, 1, or 2. The magnitude of the angular moment, L, can be found by

$$L=\sqrt{l(l+1)}\hbar$$

3. *Magnetic Quantum Number,* m_l

The electron also has a **magnetic quantum number**, m_l, which is related to the direction of the momentum. It can range from $-l$ to $+l$. For example, if $l=1$, then m_l can equal $-1, 0$, or 1. The momentum in the z-direction is found by

$$L_z=m_l\hbar$$

(The momentum in the x- and y-directions is not definite.)

4. *Spin Quantum Number,* m_s

Finally, electrons have a **spin quantum number**, m_s, which can be either $-\frac{1}{2}$ or $+\frac{1}{2}$.

Allowed and Forbidden Transitions

When an electron changes energy states, it the change in orbital quantum number needs to be one unit. So, for each transition,

$$\Delta l=\pm 1$$

If a transition follows this rule, it is an **allowed transition.** If the transition does not, it's a **forbidden transition.**

Pauli Exclusion Principle

These four quantum numbers describe the state of the electron in the atom, and can be given as (n, l, m_l, m_s). The **Pauli exclusion principle** states that no two electrons in an atom can be in the same state. Electrons in a neutral atom in the ground state fill the low energy states, and the higher energy states will be filled as more electrons are added for atoms with higher atomic numbers, Z.

Electrons with the same quantum number n are in the same **shell**. Electrons with the same quantum numbers n and l are in the same **subshell.** Letters can be used to represent the l value. Electrons with $l=0$ are in the s subshell; $l=1$ are in the p subshell; $l=2$ is the d subshell; $l=3$ is the f subshell; higher values of l follow the alphabet as g, h, i ... The electrons present in an atom in the ground state can then be represented by

$$1s^2 2s^2 2p^6 3s^1 \ldots$$

where the principal quantum number n is represented by the number, the orbital quantum number l is represented by the letter, and the number of electrons in that subshell are given by the superscript number. The maximum number of electrons that can be in a given subshell is found by $2(2l+1)$.

Chapter 30

The Nucleus

The nucleus of an atom is made up of two types of **nucleons**, protons and neutrons. The **proton** has mass $m_p=1.67262\times10^{-27}kg$ and has a positive charge, while the **neutron** has a mass $m_n=1.67493\times10^{-27}kg$. The **atomic number** is designated by Z and is the number of protons in the nucleus. The **atomic mass number** is designated by A and is the total number of nucleons in the nucleus. The number of neutrons can be found by $N=A-Z$. Each element is also designated a chemical symbol X which depends on the number of protons. A symbol can then be made for each element:

$$^{A}_{Z}X$$

Elements with the same number of protons but different number of neutrons are **isotopes** of each other.

The radius of the nucleus can be found by the relationship

$$r = (1.2\times10^{-15}m)(A^{\frac{1}{3}})$$

The nuclear mass uses for its unit the **atomic mass unit** (u) where $1u = 1.66054\times10^{-27}kg = 931.5MeV/c^2$.

Binding Energy

The total mass of individual protons and neutrons is more than the total mass if those protons and neutrons are in the nucleus of a stable atom. This is due to the **binding energy** of the nucleus. When an atom with higher binding energy is created, the change in mass is released in the form of energy, either radiation or kinetic energy. If an atom with lower binding energy is created, energy must be put into the system to create the new atom.

The binding force is created by the **strong nuclear force**, which acts over a very short distance. The protons in the nucleus of an atom are attracted by the strong nuclear force, but repelled by the electric force. As more protons and neutrons are added into a nucleus, the size of the nucleus increases, and the repulsive electric force can become stronger than the attractive strong nuclear force for protons that are farther apart in the nucleus. To counteract this effect, more neutrons will be necessary in order to contribute more strong nuclear force, which is why heavier stable elements have higher ratios of neutrons.

Radioactivity

The nucleus of an atom can decay to other elements or states. In this decay, they release high energy particles or electromagnetic radiation. Three types of radioactive decay are alpha (α), beta (β), and gamma (γ) decay. The atom that undergo radioactive decay are called the **parent**, while the resulting atom that is created after the decay is called the **daughter**.

Alpha Decay

An alpha particle is made up of two protons and two neutrons. It is essentially a high energy helium nucleus $(^{4}_{2}He)$. When an atom undergoes alpha decay, it releases a high energy alpha particle, changing its element and atomic number. A decay can be written as

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

The atomic mass goes down by 4 from the parent to the daughter, while the atomic number goes down by 2. The general form can be written as

$$^{A}_{Z}N \rightarrow {}^{A-4}_{Z-2}N' + {}^{4}_{2}He$$

The total energy released can be found by finding the change in mass from the parent isotope to the daughter particles. The total energy released is called the $Q - value$ of the decay, and is related to the initial energy of the parent particle by

$$M_P c^2 = M_D c^2 + m_\alpha c^2 + Q$$

The $Q - value$ will give the change in kinetic energy created by the decay, so it has to be a positive number in order for the decay to occur spontaneously. The $Q - value$ can be found by

$$Q = (M_P - M_D - m_\alpha)c^2$$

Beta Decay

Beta decay is the release of an electron from the nucleus. When this happens, one of the neutrons is converted into a proton. This changes the atomic number, but not the atomic mass. An example would be

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e \ (a\ neutrino)$$

The general form for beta decay is

$${}^{A}_{Z}N \rightarrow {}_{Z+1}^{\ \ A}N' + e^- + \bar{\nu}_e$$

Gamma Decay

Gamma decay occurs when the nucleus decays from an excited state to a lower energy state. This typically happens after a beta decay. The atomic number and atomic mass stay the same, but an asterisk is used to indicate an excited nucleus. The general form of gamma decay is written as

$${}^{A}_{Z}N^* \rightarrow {}^{A}_{Z}N + \gamma$$

For all alpha, beta, and gamma decays, the total number of nucleons is conserved.

Half-Life and Rate of Decay

The decay of an atom is random, following a probability. If there is a sample of N atoms, the rate at which they decay depends on the relationship

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where ΔN is the number of decays in a time Δt, and λ is the **decay constant** which depends on the isotope. The solution to this differential equation is

$$N = N_0 e^{-\lambda t}$$

where N_0 is the initial number of parent isotopes, N is the number of parent isotopes left after a time t. This relationship can also be used to find the decay activity after a time t by using the relationship

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}$$

where $\left(\frac{\Delta N}{\Delta t}\right)_0$ is the initial number of decays in a time Δt.

The **half-life** of an isotope is the average time it takes for half of the parent atoms to decay. The half-life can be found by

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Chapter 31

Nuclear Reactions

When a nucleus is struck by another, they can be transformed into other elements in a **nuclear reaction**. An example reaction would be

$${}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

where the nitrogen atom is being hit by the α particle (^4_2He), turning it into oxygen with the release of a proton (^1_1H). The general reaction is written as

$$a + X \rightarrow Y + b$$

where a is a projectile hitting X, turning it into Y and releasing a particle b. The energy released or needed for the reaction to take place is found by using the Q-value. This is found by

$$Q = (M_a + M_X - M_Y - M_b)c^2$$

The Q-value can be related to the kinetic energy of the system. If $Q > 0$, the reaction is exothermic and energy is released. The daughter particles will have an increased kinetic energy equal to the Q-value. If $Q < 0$, the reaction is endothermic, and extra energy needs to be put into the system for the reaction to take place. This extra energy can be in the form of kinetic energy in the parent particles.

Nuclear Fission

Nuclear fission is when a large nucleus is impacted with a neutron, and the nucleus is split into two nuclei with about half the atomic mass of the parent. It is common to use $^{235}_{92}U$ for the reaction. The general reaction is written as

$$n + {}^{235}_{92}U \rightarrow X_1 + X_2 + Nn$$

where N is the number of neutrons necessary to conserve the nucleon number. For any reaction, the number of protons and the number of neutrons is conserved.

The neutrons that are released will have a high kinetic energy. If there is sufficient uranium atoms, those new neutrons will hit other uranium atoms, which will in turn release new neutrons to hit other atoms. This will result in a **chain reaction**.

Chapter 32

High-Energy Particles and Accelerators

Protons, neutrons and electrons are not the only elementary particles. For example, mesons are particles that have masses that are intermediate between protons and electrons. It was found that each of these particles are made up of quarks and leptons, and the forces between particles are carried by gluons and photons. This is all described by the **standard model.**

In order to probe these fundamental particles, it was found that by bombarding nuclei with high-energy particles, new particles could be created. This is **high-energy physics**, which uses **high-energy accelerators** to create particles with high energies.

Using the de Broglie wavelength of a particle, $\lambda = h/p$, it can be seen the higher the velocity of a particle, the shorter the wavelength. Shorter wavelengths can be used to probe finer details.

Accelerators that are used include:

1. Cyclotrons – Magnetic fields in two D-shaped metal cavities curve the particles that are accelerated by an electric field between the D's. When the velocity of the particle is high enough, it will leave the D. The velocity the particles move between D's is found by

$$v = \frac{qBr}{m}$$

The **cyclotron frequency**, the rate at which the particles oscillate between D's is found by

$$f = \frac{qB}{2\pi m}$$

2. Synchrotrons – Particles pass through magnets in a giant ring. High voltages between the magnets accelerate the particles around the ring, and the particles are kept in the ring by the magnets. As the particles are accelerated, they emit radiation, especially as they move in circles. This is **synchrotron radiation.**
3. Linear Accelerators – Electrons or ions are accelerated in a straight line, resulting in no synchrotron radiation.
4. Colliders – By using two accelerators move particles in **colliding beams** which are going in opposite directions, the collision energy is maximized.

Particle Exchange

The electromagnetic force between particles is due to the exchange of photons. Richard Feynman created diagrams to illustrate the exchange of photons, which is the basis of **quantum electrodynamics** (QED). This exchange of photons is brief and the photons cannot be observed (they are busy exerting the EM force between particles) so they are *virtual.* However, the photons basically carry the EM force between charged particles. In the same way, gluons carry the strong force between neutrons and protons. There are four known forces in nature, each with a corresponding particle to carry it:

Force	Particle
Electromagnetic Force	Photons
Strong Force	Gluons
Weak Force	$W^+, W^-,$ and Z^0 bosons
Gravity	Graviton (not yet observed)

Particles and Antiparticles

For each particle, there is an antiparticle. For the electron, e^-, the antiparticle is a positron, e^+. If these two particles interact with each other, they will annihilate. They will vanish, as well as any kinetic energy they have, and the energy will be released in γ rays or other particles. Some particles, such as the π^0, are their own antiparticles.

Conservation Laws

When reactions occur, there are certain quantum numbers that need to be conserved.

Baryon Numbers

Protons and neutrons have **baryon numbers** of $B = +1$. The antiparticles have baryon numbers of $B = -1$. The total baryon number needs to be conserved in any reaction, following the principle of **conservation of baryon number.**

Lepton Numbers

There are three **lepton numbers**, each corresponding to a set of particles. The **electron lepton number**, L_e, applies to the electron, electron neutrino, and their antiparticles. For the particles, e^- and ν_e, they have a positive lepton number of $L_e = +1$. Their antiparticles, e^+ and $\bar{\nu}_e$, have negative lepton numbers of $L_e = -1$.

Muons and tau leptons follow the same pattern. For muons, the particles μ^- and ν_μ have positive **muon lepton numbers** $L_\mu = +1$ while the antiparticles μ^+ and $\bar{\nu}_\mu$ have negative lepton numbers $L_\mu = -1$. For tau leptons, the particles τ^- and ν_τ have positive **tau lepton numbers** $L_\tau = +1$ while the antiparticles τ^+ and $\bar{\nu}_\tau$ have negative lepton numbers $L_\tau = -1$.

Particles	Antiparticles	L_e	L_μ	L_τ

e^-, ν_e	$e^+, \bar{\nu}_e$	+1 (antiparticles: -1)	0	0
μ^-, ν_μ	$\mu^+, \bar{\nu}_\mu$	0	+1 (antiparticles: -1)	0
τ^-, ν_τ	$\tau^+, \bar{\nu}_\tau$	0	0	+1 (antiparticles: -1)

No other particles have lepton numbers.

For reactions, the lepton numbers for each the electron, muon, and tau lepton has to be conserved.

Particle Classification

There are three broad groups of particles. The **gauge bosons** are associated with forces, and they are responsible for carrying the forces. **Leptons** do not interact with the strong force, but do with the weak nuclear and electromagnetic forces. **Hadrons** interact with strong, weak, and electromagnetic forces with the strong force dominating over short distances. Hadrons are subdivided into **baryons** with baryon numbers $B = +1$ (or $B = -1$ for the antiparticles) and **mesons** which have $B = 0$.

Strange Particles

Beyond baryon and lepton numbers, it has been found that other quantum numbers are conserved. One such quantum number is the **strange number**. **Strange particles** are bayons that have been found to conserve the total strange number in a reaction, including the kaons (K^+, K_S^0, K_L^0) which have $S = +1$, the lambda (Λ^0) and sigma ($\Sigma^+, \Sigma^0, \Sigma^-$) particles with $S = -1$, the xi (Ξ^0, Ξ^-) particles with $S = -2$, and omega (Ω^-) with $S = -3$. For each of these, there is a corresponding antiparticle. Each particle has $B = +1$, while the baryon number for the antiparticle is negative, and their reactions all conserve the baryon number.

Quarks

Hadrons are not fundamental particles, but are rather made up of quarks. There are six quarks (up, down, strange, charmed, bottom, top), each with a baryon number of $B = \frac{1}{3}$. They have different charges on them, either $Q = -\frac{1}{3}e$ or $+\frac{2}{3}e$. The up and down quarks have $S = 0$, while the strange quark has $S = -1$.

Baryons are made up of three quarks, with a total baryon number of $B = 1$. Mesons are made from a quark and an antiquark pair, resulting in a baryon number of $B = 0$. All the different hadrons can be created by a combination of the six different quarks and antiquarks, resulting in their different quantum numbers.