This is a study guide for Exam 1. You are expected to understand and be able to answer mathematical questions on the following topics.

Chapter 23

Electric Force

The electric force is a vector quantity directed between charges. $\hat{\boldsymbol{r}}_{\boldsymbol{12}}$ is a unit vector directed from q_1 to q_2. $\vec{\boldsymbol{F}}_{\boldsymbol{12}}$ is read as "the force exerted by charge 1 on charge 2" where

$$\vec{\boldsymbol{F}}_{\boldsymbol{12}} = k_e \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}}_{\boldsymbol{12}}$$

If q_1q_2 is positive, because both charges have the same sign, the charges repel ($\vec{\boldsymbol{F}}$ is positive in the $\hat{\boldsymbol{r}}_{\boldsymbol{12}}$ direction). If q_1q_2 is negative, they attract. The vector $\hat{\boldsymbol{r}}_{\boldsymbol{12}}$ is relative between the particles so it can have both $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ quantities.

The net force on an object due to multiple charges is found with the vector sum:

$$\vec{\boldsymbol{F}}_{\boldsymbol{1}} = \vec{\boldsymbol{F}}_{\boldsymbol{21}} + \vec{\boldsymbol{F}}_{\boldsymbol{31}} + \vec{\boldsymbol{F}}_{\boldsymbol{41}} \ldots = \sum_{\boldsymbol{i}} \vec{\boldsymbol{F}}_{\boldsymbol{i1}}$$

Electric Field

The **electric field** is the force acting on a test charge *per unit charge* (units of N/C).

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_{\boldsymbol{e}}}{q_0} = k_e \frac{q}{r^2} \hat{\boldsymbol{r}}$$

Because the vector forces due to multiple charges add, so do the electric fields.

$$\vec{\boldsymbol{E}} = k_e \sum_{\boldsymbol{i}} \frac{q_i}{r_i^2} \hat{\boldsymbol{r}}_{\boldsymbol{i}}$$

The electric field due to a continuous charge distribution is found by

$$\vec{\boldsymbol{E}} = k_e \int \frac{dq}{r^2} \hat{\boldsymbol{r}}$$

Charge can be distributed across a volume, surface, or in a line:

	Volume	**Surface**	**Line**
Uniform density	$\rho = \frac{charge}{volume} = \frac{Q}{V}$	$\sigma = \frac{charge}{area} = \frac{Q}{A}$	$\lambda = \frac{charge}{length} = \frac{Q}{l}$
Non-uniform distribution	$dq = \rho dV$	$dq = \sigma dA$	$dq = \lambda dl$

A charge particle in an electric field has acceleration

$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}}{m} = \frac{q\vec{\boldsymbol{E}}}{m}$$

Chapter 24

Electric Flux

The **electric flux** is the number of electric field lines penetrating a surface (units of $N \cdot m2/C$), found by

$$\Phi_{E,i} = EA\cos\theta = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}}$$

Where $\vec{\boldsymbol{A}}$ is a vector perpendicular to the surface with a magnitude of the area. For a continuous, closed surface, Gauss's Law states

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{q_{in}}{\epsilon_o}$$

where q_{in} is the charge inside the closed surface. Gauss's Law can be used to find the electric field due to charges if an appropriate surface is chose. Try to create surfaces such that $\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = EdA$ or 0, and where E is constant over the surface so it can be removed from the integral.

If q_{in} is a distribution, you need to integrate over the region that is enclosed.

$$q_{in} = \int dq$$

where dq is one of the distributions listed in the table above.

Charges on Conducting Materials
In a conducting material, 1) the electric field is zero everywhere inside, 2) the charge resides on the surface, 3) the electric field just outside the surface is perpendicular to the surface and has a magnitude of σ/ϵ_o. If a charge is present inside a cavity of a conductor, only then will there be charges on the inner cavity surface. The charge on the inner cavity surface will have the same magnitude as the charge inside the cavity, but opposite in sign.

Chapter 25

Electric Potential Energy
The work done by a field on a charge is

$$W = \vec{\boldsymbol{F}}_{\boldsymbol{e}} \cdot \vec{\boldsymbol{s}} = q_o \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{s}}$$

If the field moves a charge, the change in **potential energy** of the charge-field system is

$$\Delta U = -W$$

(negative because the work is being done *by the field*). Generalized, the total energy change due to a field moving a charge from point A to point B is

$$\Delta U = -q_o \int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}}$$

regardless of the path taken.

Electric Potential
The **electric potential** is

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}}$$

where $d\vec{\boldsymbol{s}}$ is the displacement between two points rather than the movement of a charge. The electric potential unit is a volt (V). The electric field lines always point in the direction of decreasing electric potential.

If the electric field is constant and uniform (like in a capacitor),

$$\Delta U = q_o \Delta V = -q_o \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{s}} = -q_o Ed$$

where d is the distance moved in the direction of the E field. The E field can thus be found by finding the change in potential between two points, and dividing by the distance. The conversion is

$$1\frac{N}{C} = 1\frac{V}{m}$$

Electric Potential around Point Charges

We can define any point as the zero point for the electric potential or potential energy. If we set V=0 at $r = \infty$ for a point charge, then

$$V = k_e \frac{q}{r}$$

around point charges.

For a group of point charges,

$$V = k_e \sum_i \frac{q_i}{r_i}$$

This is *not* a vector.

To find the potential due to a charge distribution,

$$V = k_e \int \frac{dq}{r}$$

The potential energy for a test charge q_2 near a source charge q_1 would be

$$U = qV = k_e \frac{q_1 q_2}{r_{12}}$$

This is the work needed to bring a charge from infinity to a radius r_{12}. If multiple charges are present, the potential energy in the system is the work to bring each charge in one at a time. The total potential energy in the system is thus the sum of the energy of each charge pair. For example, if a third charge is brought into the system, the total potential energy would be

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

Electric field

The electric field can be found from the electric potential by

$$E = -\nabla V = -\frac{dV}{dx}\hat{x} - \frac{dV}{dy}\hat{y} - \frac{dV}{dz}\hat{z}$$

or, for a point charge,

$$E_r = -\frac{dV}{dr}$$

Charged conductor

The electric potential in a charged conductor is constant everywhere inside and on the surface. The magnitude can be found by finding the potential at the surface by using the electric field at the surface.