This is a study guide for Exam 3. You are expected to understand and be able to answer mathematical questions on the following topics.

Chapter 29

Moving Charges in a Magnetic Field

If a charge is stationary in a magnetic field, the magnetic field will have no effect on it. However, if the charge moves, the magnetic field will create a force equal to

$$\vec{\boldsymbol{F}}_{\boldsymbol{B}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

The direction of the force depends on the sign of the charge. If possible, it is easiest use the right-hand rule to find the direction of the force. The magnitude can then be found with

$$F_B = |q|vB\,\sin\theta$$

You can also use a determinant to find the magnitude and direction of $\vec{\boldsymbol{F}}_{\boldsymbol{B}}$.

The units of a magnetic field are in **tesla** (T) where

$$1T = 1\frac{N}{C \cdot m/s} = 1\frac{N}{A \cdot m}$$

If the moving charge is initially moving perpendicular to the magnetic field, it will travel in a uniform circle and the magnetic field applies a force of constant magnitude qvB. (If the magnetic field is into the page, a positively charged particle will revolve counter-clockwise, while a negatively charged particle will revolve clockwise.) We can thus say, for the charge moving in a circle,

$$F_B = qvB = ma = \frac{mv^2}{r}$$

Rearranging, we find the radius of the circular path the charged particle takes is

$$r = \frac{mv}{qB}$$

The angular speed of a particle moving in a circle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

The period of the motion would then be

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Electric and Magnetic Field Combination

The total force acting on a charge can be from both a magnetic field and an electric field at the same time, so the total force would be

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}} + q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$$

You can accelerate a charged particle with an electric field, and change the direction of motion with a magnetic field. The electric fields and magnetic fields can be used in various combinations for experiments and tools such as velocity selectors, mass spectrometers, and cyclotrons.

Magnetic Force on a Current-Carrying Conductor

If charges are moving through a wire, and that wire is in a magnetic field, the wire will experience a force of

$$\vec{\mathbf{F}}_{\mathbf{B}} = \mathrm{I}\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

where $\vec{\mathbf{L}}$ is a vector that points in the direction of the current I and has a magnitude equal to the length of the wire inside the magnetic field $\vec{\mathbf{B}}$.

Torque on a Current Loop

If a coil with N loops with a current is in a magnetic field, it will experience a torque that can be found by

$$\vec{\boldsymbol{\tau}} = NI\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$$

where $\vec{\mathbf{A}}$ has a magnitude equal to the area in the loop, and the direction is found by using the right-hand rule by following the current around the loop. The magnetic field creates a torque, unless $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel or anti-parallel, in the direction to make $\vec{\mathbf{A}}$ parallel to $\vec{\mathbf{B}}$. When these vectors are anti-parallel, the system has the highest energy because the coil has the farthest to go to make the vectors parallel. The energy of the system can be found by

$$U = -NI\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$$

The energy is thus a maximum when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are anti-parallel, and a minimum when they are parallel.

Chapter 30

Biot-Savart Law

These properties are expressed by the **Biot-Savart Law**:

$$d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}}}{r^2}$$

where $\hat{\boldsymbol{r}}$ is the direction between where the current is and a point in space, r is the distance between the place where the current is and that same point in space, and μ_0 is the **permeability of free space,**

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$

This is the little bit of magnetic field that is created by a little bit of current. The Bio-Savart Law can be used to find the magnetic field around a current-carrying wire with any length. It would be good to know the equations derived by the Biot-Savart Law for the magnetic field around a wire due to a current in that wire.

Magnetic Force Between Two Parallel Conductors

If two wires with currents are placed near each other, they exert a force on each other due to the magnetic fields they create. If we have wires 1 (test wire) and 2 (source wire), the force that wire 2 exerts on wire 1 is found by

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

where l is the length of the wires, a is the distance between them, and I_1 and I_2 are the currents in the respective wires. While this gives you the magnitude of the force, the direction of the force depends on the direction of the currents. When the currents are in opposite directions, the wires repel each other. When the currents are in the same direction, they attract each other.

Right-Hand Rule for Magnetic Fields Around a Current-Carrying Wire

It is conventional to use the right hand rule to find the direction of the magnetic field around a current-carrying wire. By pointing the thumb in the direction of the current, the fingers curl around the wire in the direction of the magnetic field.

The magnetic field forms a complete circle, and the magnetic field is constant everywhere at a radius r away from the wire.

Ampere's Law

If you follow a path around a current, you can use Ampere's Law to find the magnetic field in that path. If we say $d\vec{\boldsymbol{s}}$ is a little bit of distance around the magnetic field line's path, we can use $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$ and around the path to get the magnitude of the magnetic field:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \mu_0 I$$

where I is the total steady current passing through any surface bounded by the closed path. Ampere's Law can be used to find the magnetic field in a solenoid or toroid, both of which would be good to know for the test. As an example, forr a solenoid with N turns, we can follow a path inside the solenoid with length l:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = Bl = \mu_0 NI$$

Therefore, the magnetic field inside a solenoid is

$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$

For a circular toroid with radius a, this would become

$$B = \mu_0 \frac{N}{l} I = \mu_0 \frac{N}{2\pi a} I$$

Gauss's Law in Magnetism

Magnetic flux is similar to electric flux, where you look at the amount of magnetic flux through an area dA. The magnetic flux is then

$$\Phi_B = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$$

For a plane with area A, this becomes

$$\Phi_B = BA\cos\theta$$

where θ is the angle between $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{A}}$. The units of magnetic flux is a weber where $1\ Wb = 1\ T \cdot m^2$.

Chapter 31

Faraday's Law of Induction

If the magnetic flux through a wire loop changes, it will induce a current in the wire. This is due to an *induced emf* $\mathcal{E}$:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where $\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$. If the primary coil has N loops with area A, and each creates a magnetic flux Φ_B, the induced emf would just be the sum of the magnetic fluxes created by the loops:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

The magnetic flux through a loop is then

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

so a current can be induced by changing the magnitude of the magnetic field, the area enclosed, the angle between $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{A}}$, or a combination.

Motional emf

Motional emf is the emf induced in a conductor due to the conductor moving in a constant magnetic field. If a conducting rod moves sideways through a magnetic field, the electrons will accumulate on the end according to the magnetic force, leaving a net positive charge on the other end. The induced emf can be found by

$$\mathcal{E} = -Blv$$

where l is the length of the rod, B is the magnetic field in which the rod is moving, and v is the velocity of the moving rod. This emf can create a current in the circuit where

$$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

If the rod moves with constant velocity, the applied force to move the rod will equal the magnetic force $F_B = IlB$ that would slow the rod down. The power delivered by the applied force is then

$$P = F_{app}v = (IlB)v = \frac{B^2l^2v^2}{R} = \frac{\mathcal{E}^2}{R}$$

This shows the power input to keep the rod moving is equal to the rate at which energy is delivered into the circuit.

Lenz's Law

Lenz's law states that "the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop." This means the induced current creates a magnetic field that acts to keep the magnetic flux through the current loop a constant.